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SUSY Unification with Smooth Threshold Behavior

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Abstract

Going beyond the theta function approximation we discuss supersymmetric unification of gauge couplings with exact decoupling of light and heavy particles at energy scales below their masses. We find that the Minimal SUSY model is strongly disfavored while the Missing Doublet Model survives with GUT scale masses rising into the 10^{18} GeV region.

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Although there are tantalizing indications that the elementary forces become approximately supersymmetric in the TeV region and grand unified at a scale above 10^{16} GeV, there remain two possible problems that have recently received some attention. Problem number one is the following. The strong coupling constant at the Z scale, $\alpha_3(M_Z)$, predicted in supersymmetric (SUSY) grand unified theory (GUT), while agreeing with the apparent value at the Z resonance, disagrees markedly with the value expected from low energy measurements which lies 12% to 20% lower (3 to 5 σ). The difference between the apparent value at the Z and the actual value is one of the possible indications of a light gluino [1] and can be correlated with the excess in $Z \to b\bar{b}$ [2–4]. Problem number two lies in the fact that the scale at which the three gauge couplings appear to unify, $\sim 2 \cdot 10^{16}$ GeV, lies about two orders of magnitude below what would be expected in a more fundamental picture such as string theory. It is possible that the solution to both problems lies in physics at energy scales far above that of present accelerators. GUT scale effects are at present highly model dependent. One approach has been to assume that, at the scale at which the $U(1)$ and $SU(2)$ couplings meet, the $SU(3)$ coupling deviates from these by a fractional amount ϵ as a parametrization of gravitational or mass-splitting effects among the GUT scale particles. In such an approach the three couplings would not be seen to meet. A five percent effect at the GUT scale can lead to a ten percent difference in the predicted $\alpha_3(M_Z)$ [5–7]. A suggested approach to the second problem [8–10], is to seek an intermediate scale at which new particles lie which could redirect the gauge couplings to a unification nearer the Planck scale.

In order to explore alternative solutions to the above problems, we have, in a recent paper [11], proposed the following point of view. We assume that gravitational effects can be neglected. With this assumption of negligible gravitational effects, the model dependence of GUT scale effects lies in the unknown degeneracy splitting among GUT scale particles as well as the possibility of higher GUT scale Higgs representations. In the minimal supersymmetric model (MSSM), the GUT scale masses are M_V for the GUT scale gauge boson supermultiplet and M_Σ and M_D for the GUT scale Higgs supermultiplets. If the theory is unified far above

these masses, it will remain unified (in the theta function approximation) down to the maximum GUT scale mass below which calculable deviations from unification will occur depending on the three GUT scale masses. The full range of possible GUT scale effects in this model is then determined by running over all possible values for M_V, M_Σ, and M_D consistent with proton decay and other experimental constraints. In the Missing Doublet Model (MDM) [12] which has a richer GUT scale Higgs spectrum, there are four masses to be varied, M_V, M_Σ, M_D, and M_Φ. Above the highest GUT scale particle, the MSSM is asymptotically free while the unified coupling constant in the MDM grows with energy. This leads to a possible new constraint in the MDM, namely that the gauge coupling not become grossly non-perturbative as one approaches the Planck scale. It might be considered an appealing feature of the MDM that the coupling constant grows above the GUT scale becoming strong near the Planck scale where unification with gravity might then be natural. The result of this exercise [11] was that GUT scale degeneracy splitting in the MSSM does not solve either of the two problems mentioned above. On the other hand the MDM does lead to a value of $\alpha_3(M_Z)$ in good agreement with expectations from low energy data. This latter result is suggested in the earlier work of [13] and is also noted in [7]. For definiteness we adopt a low energy value for $\alpha_3(M_Z)$ spanning the range between two recent low energy analyses [14,15]. Thus we take

$$\alpha_3(M_Z) = \begin{cases} .104 \pm .005 & \text{(heavy gluino case)} \\ .119 \pm .005 & \text{(light gluino case)} \end{cases} \tag{1}$$

The lower extremes of this range correspond to the analysis of [14] while the upper extremes correspond to that of [15]. Many other low energy analyses are consistent with these ranges and virtually all (except for some from τ decay) lie within two σ which is sufficient for our present considerations. Another result in the MDM was that the GUT scale, defined as the maximum mass of GUT scale particles, can in fact increase into the 10^{17} GeV region [11].

In the current work we wish to explore the further refinement of smooth threshold behavior at the SUSY and GUT scales. Recently these smooth threshold effects have been given

some renewed attention [7,16]. The first of these considers smooth thresholds at the SUSY scale only, relying on an arbitrary shift of α_3 to parameterize GUT scale effects. The second does not discuss the solution ranges for the GUT scale masses. In the region of overlap our results are consistent with theirs and we extend their conclusions.

It is well known that the variation in a gauge coupling $\alpha(q)$ due to the one-loop propagator correction from a fermion of mass m_i in the Euclidean region is given [17] by

$$4\pi q\frac{d}{dq}\alpha^{-1}(q) = -2b_i n_f(q/m_i) \tag{2}$$

where

$$n_f(q/m) = 6\int_0^1 \frac{dx x^2(1-x)^2}{x(1-x)+m^2/q^2} = 1-\frac{3}{2}(w^2-1)\left[1-\frac{w^2-1}{2w}\ln\left(\frac{w+1}{w-1}\right)\right] \tag{3}$$

with

$$w = \sqrt{1+4m^2/q^2}. \tag{4}$$

For a bosonic loop one would have

$$4\pi q\frac{d}{dq}\alpha^{-1}(q) = -2b_i n_b(q/m_i) \tag{5}$$

with

$$n_b(q/m) = 1+3(w^2-1)\left[1-\frac{w}{2}\ln\left(\frac{w+1}{w-1}\right)\right]. \tag{6}$$

$n_f(q/m)$ and $n_b(q/m)$ exhibit the decoupling behavior,

$$n_{f,b}(q/m) \rightarrow \begin{cases} 1 & q \gg m \\ 0 & q \ll m \end{cases} \tag{7}$$

So that, as an approximation, one can write for the contribution from a particle of mass m as

$$n_{f,b}(q/m) = \theta(q-m) \qquad \text{(theta function approximation).} \tag{8}$$

The n's can be written analytically as perfect derivatives

$$n_{f,b}(q/m) = q\frac{d}{dq}f_{f,b}(q/m) \tag{9}$$

with

$$f_f(q/m) = \frac{w^2}{2}\left[1 - \frac{w^2-3}{2w}\ln\left(\frac{w+1}{w-1}\right)\right] \tag{10}$$

$$f_b(q/m) = \frac{w^2}{2}\left[-1 + w\ln\left(\frac{w+1}{w-1}\right)\right] \tag{11}$$

Thus if one integrates eq.2 from some q_0 to q_1 the result is

$$4\pi\alpha^{-1}(q_0) = 4\pi\alpha^{-1}(q_1) - 2b\left[f_f(q_0/m) - f_f(q_1/m)\right] \tag{12}$$

with the corresponding result, changing f_f for f_b, in the case of a contribution from a boson. If $q_0/m \ll 1 \ll q_1/m$, this becomes

$$\begin{aligned} 4\pi\alpha^{-1}(q_0) = 4\pi\alpha^{-1}(q_1) - 2b\Bigg[&-\ln(q_1/m) + \frac{5}{6} + \frac{q_0^2}{10m^2} - \frac{5}{2}\frac{m^2}{q_1^2} \\ &+ \mathcal{O}\left((q_0/m)^4, (m/q_1)^4\ln(q_1/m)\right)\Bigg] \end{aligned} \tag{13}$$

The theta function approximation is equivalent to keeping only the first term in the square bracket. The constant term can be taken into account in the theta function approximation by imposing a discrete shift (matching condition) in the couplings at $q = m$ but in practice this shift is generally neglected. If there are particles in the vicinity of q_0 or q_1 the power series in eq.13 is slowly convergent and the theta function approximation becomes poor. Of course, the theta function approximation could be defined to be exact if the non-logarithmic effects were properly incorporated elsewhere as in extracting couplings from data [18] but in practice this is not done and it seems much more economical to include the threshold mass effects into the running of the couplings. Further discussion of the theoretical basis for smooth decoupling ("Mass Dependent Subtraction Procedure") is given in [16]. Practically all current grand unification studies including [19–22,11] have relied heavily on the theta function approximation to the beta function. A notable exception has been the work of [24] where the full $n_f(q/m)$ was used for the top quark and gaugino contributions together

with the full $n_b(q/m)$ for the SUSY scalars. However even in this work the effect of smooth thresholds at the GUT scale was neglected. In the current work we extend the smooth threshold behavior to the GUT scale particles and to the low-lying quarks, leptons, and gauge bosons. Our purpose is to investigate the differential effect on the gauge unification solutions when one goes from theta function to smooth decoupling. The effect on the b/τ mass ratio, which is not part of the current study, would be expected to be especially significant since the b Yukawa which, in the theta function approximation, is rising most rapidly in the low energy region begins to be strongly suppressed as one approaches the b scale if one imposes a smooth decoupling. The GUT scale effects are also expected to be large. Therefore, we do not attempt to fit the top quark mass, $\tan\beta$ or the b/τ mass ratio.

In [11] we considered, in the theta function approximation, the effect of non-degeneracy among the GUT scale particles in both the minimal supersymmetric model and the missing doublet model. In such a treatment the GUT scale is considered to be the mass of the heaviest of the GUT scale particles since if the couplings are unified there they remain unified at higher energies. With smooth decoupling the couplings can be assumed to be unified far above the GUT scale masses but will begin to diverge as one approaches the GUT scale. We somewhat arbitrarily take unified couplings at the Planck mass assuming that all GUT scale particles have much smaller masses and that perturbation theory is still at least qualitatively valid there. Gravitational effects, which are beyond the scope of the present paper, are supplementary to the effects studied here but should not void our qualitative conclusions. In this paper we restrict our interest to gauge coupling unification. We integrate the one-loop contributions exactly and analytically including the full threshold behavior while treating the two loop (including Yukawa) contributions, δ_i^{2L}, numerically with a crude (but fully adequate) approximation to smooth decoupling ignoring the two loop contributions of the GUT scale particles. The two loop contribution, δ_3^{2L} is found to be only 5 to 7 percent of the analytic one loop contribution in the MDM and 9 to 17 percent in the MSSM and roughly linearly related to $\alpha_3(M_Z)$ in each case. The b coefficients are as given in [13,11] except that we separate the GUT scale Higgs supermultiplet contributions

into separate contributions from bosons and fermions in the ratio of 1:2. Similarly the contribution of the GUT scale gauge supermultiplet separates into bosons and fermions in the ratio 11:(-2). The top Yukawa, $\alpha_t(M_P)$, at the gauge unification point is taken to be between 0.1 and 0.9. Due to the fixed point behavior large values of the top Yukawa rapidly evolve down to values of order α_3 so that the gauge couplings are relatively insensitive to the GUT scale behavior and values of the Yukawa couplings. Then

$$4\pi\alpha_i^{-1}(M_Z) = 4\pi\alpha_i^{-1}(M_P) - 2\sum_j b_i(j)\left[f_j(M_Z, m_j) - f_j(M_P, m_j)\right] + \delta_i^{2L}. \tag{14}$$

At the Planck mass, $1.22 \cdot 10^{19}$ GeV, we take the unification condition

$$4\pi\alpha_3^{-1}(M_P) - 1 = 4\pi\alpha_2^{-1}(M_P) - \frac{2}{3} = 4\pi\alpha_1^{-1}(M_P) \equiv 4\pi\alpha_0^{-1}. \tag{15}$$

We choose the Planck scale gauge coupling, α_0, and the Planck scale top Yukawa, $\alpha_t(M_P)$, at random as we do for the various GUT scale masses, $M_V, M_D, M_\Sigma, M_\Phi$, and the SUSY scales, m_0, and $m_{1/2}$. For perturbative consistency, however, we require that $\alpha_0 < 1/4$, $\alpha_t(M_P) < 1$, and $m_j/M_P < 1/5$. The SUSY masses are chosen with 100 GeV $< m_0 <$ 1 TeV and 50 GeV $< m_{1/2} <$ 330 GeV. Splitting between partners of left and right handed fermions is neglected. The squark and slepton masses are defined in terms of m_0 and $m_{1/2}$ as in [16]. That is: $m_{\tilde{q}}^2 = m_0^2 + 7m_{1/2}^2$, $m_{\tilde{l},L}^2 = m_0^2 + 0.5m_{1/2}^2$, and $m_{\tilde{l},R}^2 = m_0^2 + 0.15m_{1/2}^2$. We discard as non-solutions values of these parameters inconsistent with the renormalization group running and the experimental values $\alpha^{-1}(M_Z) = 127.9 \pm .2$ and $\sin^2\theta_W = .2320 \pm .0008$. Further technical details on our "top-down" approach may be found in [11]. By such Monte Carlo methods it is possible to completely determine the multidimensional solution space.

In [11] it was found that the MSSM, with or without GUT scale degeneracy breaking, was inconsistent with the low energy data on α_3 if proton decay constraints and a theoretically desirable SUSY scale below 1 TeV were imposed. This was also noted in the degenerate case by [25] and has been emphasized more recently by [4]. The lower limits on $\alpha_3(M_Z)$ in the MSSM with theta function decoupling are consistent with those found by other authors [26,23]. If one requires only agreement with the LEP values of α_3 the inconsistency is

not apparent. One of the conclusions of [11] was that this inconsistency disappears if the MSSM is replaced by the missing doublet model. This result was also noted in [7]. These considerations are independent of whether or not the gluino is light (in the GeV region) as is still not experimentally ruled out. In table 1 we compare the smooth threshold results for $\alpha_3(M_Z)$ and the GUT scale masses with the results of [11] where GUT scale degeneracy was broken but sharp (theta function) decoupling was used. Slight differences between our current requirements and those of [11] with regard to $\sin^2\theta_W$ and the b/τ mass ratio do not affect the clear, qualitative conclusions that can be drawn from the comparison in table 1. The numbers in table 1 are shown in the heavy gluino scenario but are not sensitive to this choice. For example, with smooth decoupling, if one puts $m_{1/2} = 0$ (light gluino option) the minimum $\alpha_3(M_Z)$ drops only to 0.113 in the MDM and only to 0.169 in the MSSM. On the other hand Ref. [7] finds some preference in the MSSM for $m_{1/2} \ll m_0$ implying at least a relatively light gluino. However, our results indicate that with smooth decoupling at the GUT scale the MSSM cannot be saved by this mechanism. In the MDM we discard solutions with $\alpha_3(M_Z)$ above 0.135 since these seem of no phenomenological interest. The same requirement in the MSSM would eliminate all solutions leading to our conclusion that the MSSM is no longer viable when smooth decoupling is taken into account.

To summarize the conclusions of this study we may say the following. In [11] we noted that the MSSM with theta function threshold behavior predicted an $\alpha_3(M_Z)$ inconsistent with extrapolations from low energy data. Our current results strongly reinforce this conclusion and disfavor the MSSM even if the higher LEP values of $\alpha_3(M_Z)$ are used. Because of the very large values of α_3 predicted in the MSSM with smooth thresholds, current estimates of gravitational effects cannot salvage the situation without calling into question the successful prediction of $\sin^2\theta_W$. For this reason it is our opinion that the MSSM is highly unlikely to be realized in nature. The low energy measurements of α_3 and $\sin^2\theta_W$, therefore, strongly suggest a richer GUT scale Higgs structure such as that given in the MDM. The phenomenological superiority with respect to grand unification of the MDM over the MSSM was first pointed out in [11]. This model, when smooth threshold behavior is taken into

account, also contains unification solutions with the heaviest GUT scale particles in the 10^{18} GeV region as suggested by string theory. If we require $\alpha_3(M_Z) < 0.12$ we find, in fact, that all the solutions in the MDM have $M_\Phi > 5 \cdot 10^{17}$ GeV. All the MDM solutions have the leptoquark gauge boson supermultiplet in the 10^{16} GeV region or below suggesting that proton decay could be dominated by the ep decay modes expected in non-supersymmetric SU(5). If we compare the unification lower limits on $\alpha_3(M_Z)$ from table 1 with the results from low energy analyses given in eq.1 we see that the light gluino option is somewhat favored. However, if there are 10% effects from gravity or other sources this preference might be eliminated.

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TABLES

TABLE I. Minimum and maximum values of $\alpha_3(M_Z)$ and the GUT scale masses in the unification solution space of the MSSM and MDM with either sharp or smooth decoupling. Underlined values are upper or lower limits imposed for phenomenological reasons discussed in the text.

	MSSM sharp	MSSM smooth	MDM sharp	MDM smooth
$\alpha_3(M_Z)$	$(.117, .133)$	$(.174, .24)$	$(.095, .114)$	$(.116, \underline{.135})$
M_V (GeV)	$(0.7, 82)10^{15}$	$(1.8, 130)10^{15}$	$(2.3, 21)10^{15}$	$(\underline{1.0}, 6.4)10^{15}$
M_D (GeV)	$(\underline{1.0}, 24)10^{16}$	$(\underline{.01}, \underline{2.4})10^{18}$	$(\underline{1.0}, 18)10^{16}$	$(\underline{1.0}, 24)10^{16}$
M_Σ (GeV)	$(0.3, 15)10^{16}$	$(\underline{.01}, \underline{2.4})10^{18}$	$(\underline{1.2}, 6.7)10^{16}$	$(.021, \underline{2.4})10^{18}$
M_Φ (GeV)	-	-	$(\underline{1.0}, 7.8)10^{16}$	$(0.21, \underline{2.4})10^{18}$